FAST ALGORITHM FOR ADMITTANCE CALCULATION OF SAW TRANSDUCERS

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Abstract - Analytic formulae for calculation of the surface acoustic wave (SAW) transducer admittance comprising acoustic radiation conductance and susceptance are deduced neglecting multiple interelectrode SAW interactions (quasi-static approximation). Based on the aperture channelizing technique, an acoustic admittance is treated as a sum of the channel admittances. By applying a special summation technique for the apodized periodic SAW transducers, the general formula is converted to the recurrent form resulting in considerable reduction of computation time.

Introduction

Calculation of a SAW transducer admittance is an integral part of the computer-aided design of SAW bandpass filters. Accurate modeling is necessary to evaluate a priori SAW filter insertion loss, simulate frequency response distortion due to the electrical interaction with source and load, match SAW devices. Unfortunately, rigorous analysis techniques [1] are impracticable due to their complexity and computation slowness. Usually, the equivalent circuit model of the uniform SAW transducer is used [2] and aperture-channelizing technique [3-5] is applied for apodized SAW transducers. As a rule, it is radiation conductance that can be determined within SAW model and numeric Hilbert transformation is performed to calculate radiation susceptance [3].

Calculations are simplified in the quasi-static approximation [5] where the superposition principle can be applied to calculate radiation conductance of an unapodized periodic SAW transducer with an arbitrary polarity sequence [3, 4]. Unfortunately, simple analytic formulae comprising both radiation conductance and susceptance were deduced for uniform multielectrode transducers only [5].

In the present paper, analytic recurrent formulae for SAW transducer admittance calculation comprising acoustic conductance and susceptance are deduced in quasi-static approximation.

Admittance Calculation

For the calculation of the acoustic admittance $Y(\omega)=G(\omega)+jB(\omega)$, an apodized SAW transducer is subdivided into uniform (unapodized) acoustical channels as shown in Fig.1. The total transducer admittance can be determined as a sum of channel admittances

$$Y(\omega) = \sum_i Y_i(\omega), \tag{1}$$

where $Y_i(\omega)=G_i(\omega)+jB_i(\omega)$ is the elemental admittance of the i-th channel with the aperture W_i. The channel conductance $G_i(\omega)$ is given by the following expression [5]

$$G_i(\omega) = \omega W_i \Gamma \mid F_i(k)\rho(k)\mid^2 \tag{2}$$

with the channel array factor

$$F_i(k) = \sum_n P_n^i \; e^{-jkx_n}, \tag{3}$$

and the element factor

$$\rho(k) = \varepsilon \frac{2\sin \pi s}{P_{-s}(-\cos\Delta)} P_s(\cos\Delta), \qquad (4)$$

where $k=\omega/v$ – SAW wave number, v – SAW velocity; $\Gamma=k^2/2\varepsilon$ – substrate constant, k^2 – electromechanical coupling factor; $\varepsilon=\varepsilon_o+\varepsilon_p$ – effective permittivity, $s=kp/2\pi-n$ – normalized frequency variable, $n=[kp/2\pi]$ - space harmonic number, $P_{-s}(-\cos\Delta)$ - Legendre function, $P_n(\cos\Delta)$ - Legendre polynomial, $\Delta=\pi a/p$.

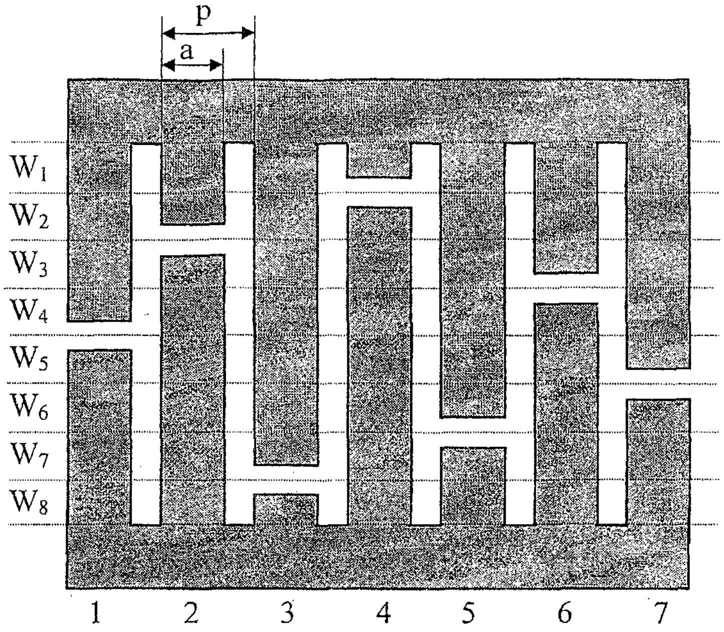


Fig.1. Aperture channelizing

The algorithm proceeds as follows. First, the coordinates y_i, $i=1,N$ of the transversal gaps of an apodized IDT comprising N fingers are lexicographically sorted in the increasing order and the transducer is subdivided into N parallel acoustic tracks (channels), with the channel boundaries given by the coordinates y_i. As two neighbour channels differ with a polarity of one finger only, one exponent of the type $exp(-jkx_i)$ is needed to recurrently compute the elemental frequency response of the channel where x_i is the x-coordinate of the finger at the channel boundary y_i. An example of the polarity sequences corresponding to different channels in Fig. 1 is given in the table.

Polarity sequences for different channels

N	1	2	3	4	5	6	7
P_n^1	+	+	+	+	+	+	+
P_n^2	+	+	+	-	+	+	+
P_n^3	+	-	+	-	+	+	+
P_n^4	+	-	+	-	+	-	+
P_n^5	-	-	+	-	+	-	+
P_n^6	-	-	+	-	+	-	-
P_n^7	-	-	+	-	-	-	-
P_n^8	-	-	-	-	-	-	-

Therefore, we obtain the following recurrent relation for the radiation conductance

$$G_i(\omega) = g(\omega)W_i \,|\, F_i(k)\,|^2, \qquad (5)$$

$$g(\omega)=\omega\Gamma|\rho(k)|^2, \tag{6}$$

$$F_i=F_{i-1}-2e^{-jkx_i}, \quad F_1=\sum_n e^{-jkx_n}. \tag{7}$$

By substitution of the Eq.(7) into Eq.(5) we obtain after transformations the recurrent formula

$$G_i(\omega)=\frac{W_i}{W_{i-1}}G_{i-1}-4W_i g(\omega)\,\mathrm{Re}\left\{F_{i-1}e^{jkx_i}\right\}+4\,W_i\,g(\omega). \tag{8}$$

The radiation susceptance can be calculated as the Hilbert transformation of the radiation conductance [5]

$$B(\omega)=H\{G(\omega)\}=-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{G(\omega')}{\omega-\omega'}d\omega'. \tag{9}$$

By applying the Hilbert transformation (9) to the Eq.(8) we deduce the following equation for the channel susceptance

$$B_i(\omega)=\frac{W_i}{W_{i-1}}B_{i-1}-4\,W_i\,\mathrm{Re}\left\{H\left\{g(\omega)F_{i-1}e^{jkx_i}\right\}\right\}+4W_i\,H\{g(\omega)\}. \tag{10}$$

Using the Sochotzky-Plemel formulae [6, 7] for the Koshi-type integral with singularity on the real axis, we derive the following closed-form relation

$$\mathrm{Re}\left\{H\left\{g(\omega)F_{i-1}e^{jkx_i}\right\}\right\}=\mathrm{Im}\left\{g(\omega)F_{i-1}e^{jkx_i}\right\}. \tag{11}$$

After closed-form integration (11) we obtain

$$B_i(\omega)=\frac{W_i}{W_{i-1}}B_{i-1}-4W_i\,\mathrm{Im}\left\{g(\omega)F_{i-1}e^{jkx_i}\right\}+4W_i\,H\{g(\omega)\}. \tag{12}$$

The last term in the Eq. (12) should be calculated by numerical integration. It depends on the metallization ratio as a parameter and can be approximated by the elemental functions.

By combining the Eqs. (8) and (12) the formula for SAW transducer admittance comprising radiation conductance and susceptance takes the recurrent form

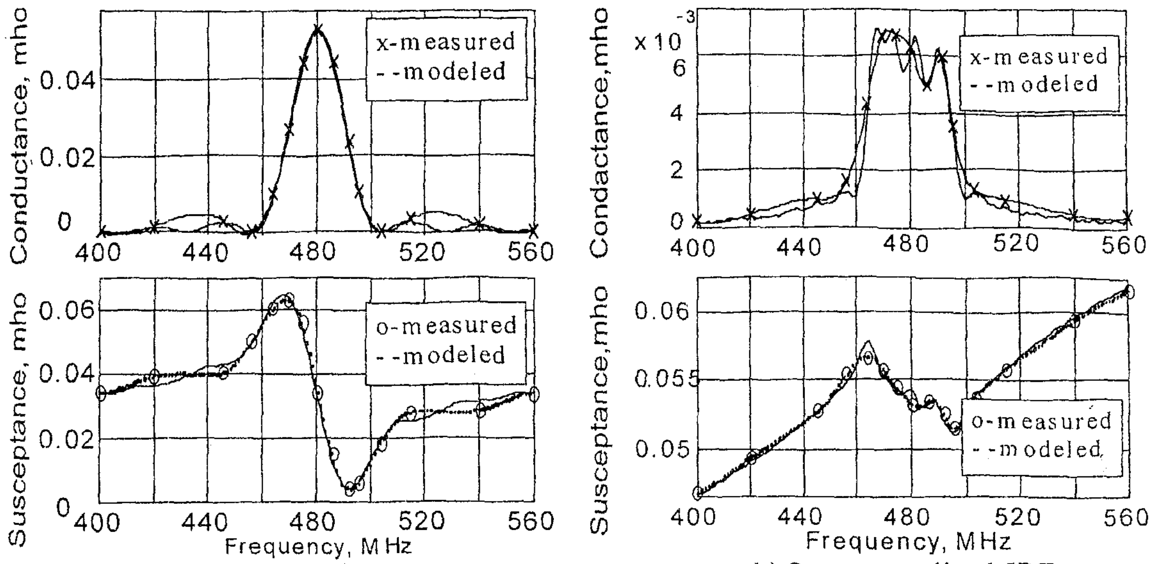
$$Y_i(\omega)=\frac{W_i}{W_{i-1}}Y_{i-1}(\omega)+\Delta Y_i(\omega), \tag{13}$$

$$\Delta Y_i(\omega)=4W_i\,g(\omega)\left(1-F_{i-1}e^{jkx_i}\right)+4j\,W_i\,H\{g(\omega)\}. \tag{14}$$

The static capacitance of an apodized SAW transducer can be found in the closed-form using the technique [9], for example.

Calculation Example and Experimental Results

An example of the modeled and measured admittance characteristics for a SAW bandpass filter is shown in Fig. 2. The SAW filter has the central frequency f_0=479.5 MHz, passband width Δf_{-3}=27 MHz, shape factor $K_{-3/-40}$=1.8. The input and output transducers with split fingers have an aperture W=340 μm, metallization ratio η=0.5, and electrode numbers N_1=60 and N_2=351, respectively. The substrate material is the $Y128^oX$ lithium niobate with electromechanical coupling factor k^2=5.5 % and permittivity $\varepsilon=55\varepsilon_0$. Input and output capacitances were calculated by applying the technique [9]. Both unapodized and apodized SAW transducer admittances were calculated using the Eq. (13). Results of the admittance calculation for SAW transducer with split fingers show good agreement with measured admittance characteristics.



a) Input unapodized IDT b) Output apodized IDT

Fig.2. Modeled and measured admittance characteristics

Conclusion

The recurrent formulae for admittance calculation of the apodized SAW transducer have been deduced in quasi-static approximation, which allow to reduce considerably the computation time. Given a sorted transversal gap coordinates, the computation time depends linearly on the finger number N and it is basically the same as the time required for the frequency response calculation using the Fourier transform. As for a particular IDT sorting is a single-time frequency-independent procedure, it does not contribute considerably to the overall computation time. Quick sorting algorithms can be applied to long transducers. To further reduce the computation time, all the N exponents involved in computation at a particular frequency can be calculated a priori and stored in memory, with the recurrent procedure comprising summation and multiplication operations only. The proposed computational algorithm can be applied to both periodic and non-periodic transducers, for example, to chirp transducers. The examples of the admittance calculation are presented which show good correspondence of the modeled and experimental results.

References

1. R.F. Milsom, N.H.C. Reilly. "Analysis of generation and detection of surface and bulk acoustic waves by interdigital transducer", IEEE Trans. Sonics and Ultrasonics, 1977, vol. SU-24, No 3, pp. 147-166.
2. W.R. Smith et al. "Analysis of interdigital surface wave transducer by use of an equivalent circuit model", IEEE Trans. Microwave Theory and Techn., 1969, vol. MTT-17, No 11, pp. 856-864.
3. S. Datta, B.J. Hunsinger and D.C. Malocha. "A generalized model for periodic transducers with arbitrary voltages", IEEE Trans. Sonics and Ultrasonics, 1979, vol. SU-26, No 3, pp. 243-245.
4. S. Datta. *Surface acoustic wave devices*. New Jersey: Prentice-Hall, Englewood Cliffs, 1986.
5. D.P. Morgan. *Surface-wave devices for signal processing*. Amsterdam: Elsevier, 1985.
6. F.D. Gahov. *Boundary problems*. Moscow: Science, 1977 (in Russian).
7. F.D. Gahov, Y.I. Chersky. *Convolution equations*. Moscow: Science, 1978 (in Russian).
8. M.A. Lavrentiev, B.V. Shabatt. *Theoretical functions of complex variable*. Moscow: Science, 1987 (in Russian).
9. A.S. Rukhlenko. "Charge distribution and capacitance calculation for generalized periodic SAW transducers using Floquet`s technique", IEEE Ultrasonics Symp. Proc. , 1994, pp. 325-329.